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Pressure Field of a Vortex Wake in Ground Effect

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THE flowfield produced by the approach of a point vortex pair to a plane boundary was presented years ago by Grobli[1] and can be found in Lamb's, *Hydrodynamics.*[2] Recently the FAA has studied the problem of locating descending vortices generated by aircraft by use of near ground measurements of both velocity and pressure.[3] To correlate the measurements with height and intensity of the vortices a theoretical base is desirable but heretofore the pressure field of the simple vortex field has not been available.

The shed vorticity of a lifting aircraft wing rolls up rather quickly and for an elliptically span-loaded wing the bulk of the vorticity shed from each side is closely centered about the points laterally situated at a distance $\pm\pi s/4$ from the aircraft's centerline; s is the wing semispan distance. Because of the slow variation in the field with distance behind the aircraft, the development of the vortex wake behind the aircraft can be approximated assuming the flow is two-dimensional. Thus for two vortices located at the points $(\pm x_0, y_0)$ with the ground plane at $y=0$, the disturbance velocity potential function can be written

$$\phi=\frac{\Gamma}{2\pi}\left[\tan^{-1}\frac{y-y_0}{x-x_0}+\tan^{-1}\frac{y+y_0}{x+x_0}-\tan^{-1}\frac{y-y_0}{x+x_0}-\tan^{-1}\frac{y+y_0}{x-x_0}\right] \tag{1}$$

where Γ is the circulation of the wing at the centerline and the coordinate system is defined in Fig. 1.

The velocity field follows by differentiation of Eq. (1);

$$u=\frac{\partial\phi}{\partial x}=\frac{\Gamma}{2\pi}\left[\frac{y_0-y}{[(x-x_0)^2+(y-y_0)^2]}-\frac{(y_0+y)}{[(x+x_0)^2+(y+y_0)^2]}+\frac{(y-y_0)}{(x+x_0)^2+(y-y_0)^2}+\frac{(y+y_0)}{(x-x_0)^2+(y+y_0)^2}\right] \tag{2}$$

$$v=\frac{\partial\phi}{\partial y}=\frac{\Gamma}{2\pi}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0)^2}+\frac{x+x_0}{(x+x_0)^2+(y+y_0)^2}-\frac{x+x_0}{(x+x_0)^2+(y-y_0)^2}-\frac{x-x_0}{(x-x_0)^2+(y+y_0)^2}\right] \tag{3}$$

The vortex velocity components u_0 and v_0 can be obtained from Eq. (2) and (3) by omitting the first terms and setting $x=x_0$ and $y=y_0$; we obtain then

$$u_0=\frac{\partial x_0}{\partial t}=\frac{\Gamma}{4\pi}\left[\frac{x_0^2}{y_0(x_0^2+y_0^2)}\right] \tag{4}$$

$$v_0=\frac{\partial y_0}{\partial t}=\frac{\Gamma}{4\pi}\left[\frac{-y_0^2}{x_0(x_0^2+y_0^2)}\right] \tag{5}$$

Elimination of time in the above equations and integration yields an equation for the path of the vortices:

$$y_0^2=\frac{(\pi s/4)^2x_0^2}{x_0^2-(\pi s/4)^2} \tag{6}$$

The disturbance pressure field is computed from the relation

$$\nabla p=-\rho\frac{\partial\phi}{\partial t}-\frac{1}{2}\rho[u^2+v^2] \tag{7}$$

in which ρ is the fluid density. Since,

$$\frac{\partial\phi}{\partial t}=\frac{\partial\phi}{\partial x_0}\frac{\partial x_0}{\partial t}+\frac{\partial\phi}{\partial y_0}\frac{\partial y_0}{\partial t} \tag{8}$$

we obtain using the derived equations the expression for the pressure on the ground plane $y=0$:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2}=\left(\frac{4C_Ls}{\pi^2A}\right)^2\left\{\frac{x_0^2(x_0^2+y_0^2+x^2)}{[y_0^2+(x-x_0)^2][y_0^2+(x+x_0)^2](x_0^2+y_0^2)}+\frac{y_0^2(x_0^2+y_0^2-x^2)}{[y_0^2+(x-x_0)^2][y_0^2+(x+x_0)^2](x_0^2+y_0^2)}-\left[\frac{4xx_0y_0}{[y_0^2+(x-x_0)^2][y_0^2+(x+x_0)^2]}\right]^2\right\} \tag{9}$$

which makes use of the relation for elliptically loaded wings

$$\Gamma=\frac{4C_LVs}{\pi A} \tag{10}$$

where C_L is the wing lift coefficient and A is the wing aspect ratio. The pressure as expressed by Eq. (9) varies in time because the vortex positions $\pm x_0, y_0$, are changing with time. The time and position relationship has been obtained by numerical integration of Eqs. (4) and (5) using Eq. (6).

The pressure distribution at various times in shown graphically in Fig. 1. It is interesting to see that as the vortices first approach the ground, only positive pressures are produced; however, as the descent continues the high velocity field of the vortices makes itself apparent in the dips to subatmospheric pressure which lie closely beneath the vortex center. Nevertheless, a substantial positive pressure hill precedes the path of the vortex as it moves laterally in the "ground effect." The level of pressures produced is dependent only on

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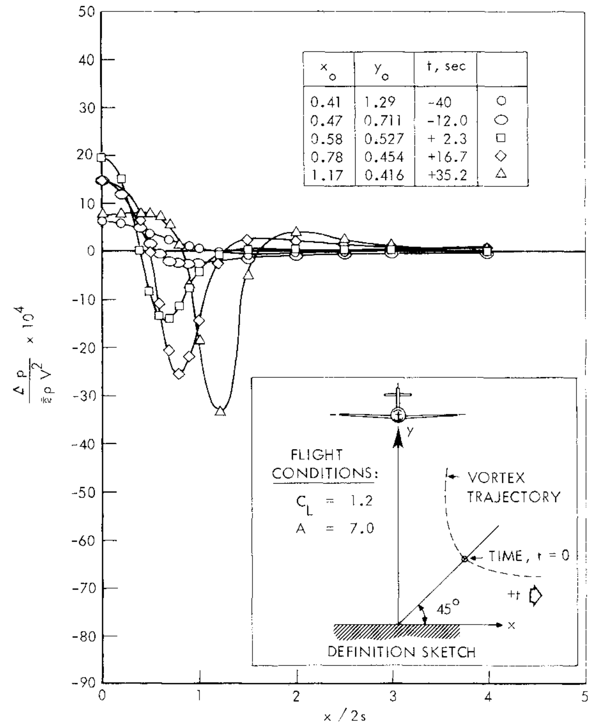

Fig. 1 Ground pressure under a symmetrical descending vortex pair.

the flight speed, lift coefficient and aspect ratio but the length scale of the pressure distribution is proportional to the span.

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Effect of Heating on Leading Edge Vortices in Subsonic Flow

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LEADING edge vortices are the primary source of lift on delta wings at high angles of attack.[1] The suction resulting from these vortices and the associated three-dimensional flow delays separation on delta wings to very high angles of attack and usually results in a relatively smooth stall for the wing. The space shuttle vehicle will employ a thick delta wing which will (like all delta wing vehicles) assume high angles of attack upon landing. A unique problem for the space shuttle, however, will occur due to the heating it will experience during re-entry. The subsequent glide and landing maneuvers will be conducted with a hot wing surface. It is therefore of interest to know how this heating from the wind surface will effect the wing's low speed aerodynamics, and in particular, the leading edge vortex.

During re-entry the shuttle wings should reach soak temperatures of 1000°F or more. To study the effects of such wing surface temperatures on a delta wing an experimental study was conducted in the Virginia Tech Six Foot Stability Wind Tunnel. An aluminum, 60° sweep, double delta wing (Fig. 1) was cast using a wood model which has been previously tested at NASA Langley Field.[2] This wing was tested in the wind tunnel using a six component mechanical balance system to avoid heating errors which would be experienced with strain gage balances. Heating was accomplished by the use of a special 70,000 BTU/hr infrared gas burner system. With this system the wing could be heated to over 600°F, making possible surface temperature to freestream absolute temperature ratios up to two.

Extensive testing was performed on the unheated wing to verify the nature of the flow and check out the balance system. Flow visualization tests showed the classic leading edge vortex formation with increasing angles of attack and a highly three-dimensional flow at higher angles.[3] Force and moment data were taken over a wide range of angle of attack and yaw angle. This data essentially duplicated that reported in Ref. 2 for the same wing.[3]

Heated wing tests were conducted by heating the wing to at least twice the absolute ambient temperature, removing the external heaters while accelerating the tunnel to the desired speed, and recording forces, moments, and wing surface temperature as the tunnel speed remained steady and wing temperature dropped. Temperature data were obtained from ten thermocouples imbedded in the wing surface. This procedure was repeated for all desired angles of attack and yaw. All tests were run at a Reynolds number of approximately 1.6 x 10,[6] representing a speed of 148 fps.

Results and Conclusions

The primary results from the heated delta wing tests are given in Figs. 2-4 as plots of lift, drag, and pitching moment coefficient vs wing surface to freestream absolute temperature ratio for various angles of attack from 0 to 36°. All data shown are for zero yaw (the results were similar for yaw angles of 6 and 10°). It is seen that temperature ratios up to two have virtually no effect on either lift or pitching moment; however, the effect on drag is strong and increases with angle of attack. This leads to the conclusion that heating has virtually no effect on the leading edge vortices for the delta wing

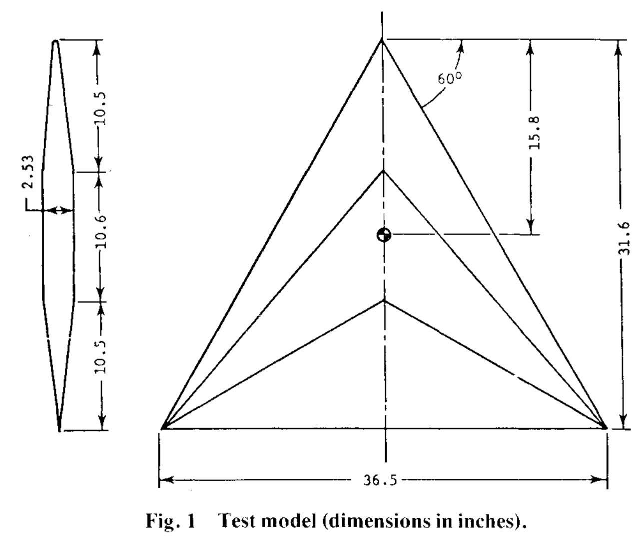

Fig. 1 Test model (dimensions in inches).

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